Edexcel Maths M3

Topic Questions from Papers

Dynamics

Leave blank

7. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A. In its equilibrium position, P is 0.85 m vertically below A.

(a) Show that $\lambda = 39.2$.

(2)

The particle is now displaced to a point B, 0.95 m vertically below A, and released from rest.

(b) Prove that, while the string remains stretched, P moves with simple harmonic motion of period $\frac{\pi}{7}$ s.

(6)

(c) Calculate the speed of P at the instant when the string first becomes slack.

(3)

The particle first comes to instantaneous rest at the point C.

(d) Find, to 3 significant figures, the time taken for P to move from B to C.

(5)

Leave blank

3. A spacecraft S of mass m moves in a straight line towards the centre of the earth. The earth is modelled as a fixed sphere of radius R. When S is at a distance x from the centre of the earth, the force exerted by the earth on S is directed towards the centre of the earth and has magnitude $\frac{k}{x^2}$, where k is a constant.

(a) Show that $k = mgR^2$.

(2)

Given that S starts from rest when its distance from the centre of the earth is $2R$, and that air resistance can be ignored,

(b) find the speed of S as it crashes into the surface of the earth.

(7)

Leave blank

5. A particle P moves on the x-axis with simple harmonic motion about the origin O as centre. When P is a distance 0.04 m from O, its speed is 0.2 m s^{-1} and the magnitude of its acceleration is 1 m s^{-2}.

(a) Find the period of the motion.

(3)

The amplitude of the motion is a metres.

Find

(b) the value of a,

(3)

(c) the total time, within one complete oscillation, for which the distance OP is greater than $\frac{1}{2}a$ metres.

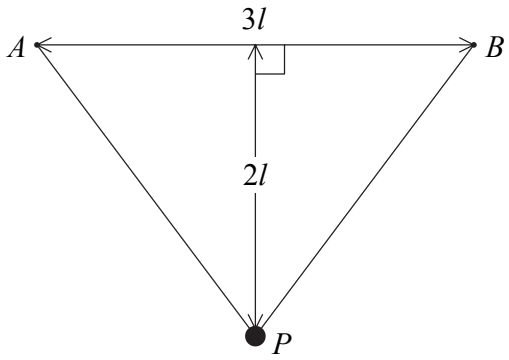
(5)

Leave blank

Question 5 continued

Leave blank

7. **Figure 1**



A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in Figure 1,

(a) show that $\lambda = \frac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(b) Show that P comes to instantaneous rest on the line AB.

(6)

Leave blank

Question 7 continued

Leave blank

2. A particle P moves with simple harmonic motion and comes to rest at two points A and B which are 0.24 m apart on a horizontal line. The time for P to travel from A to B is 1.5 s. The midpoint of AB is O. At time $t = 0$, P is moving through O, towards A, with speed u m s^{-1}.

(a) Find the value of u.

(4)

(b) Find the distance of P from B when $t = 2$ s.

(5)

(c) Find the speed of P when $t = 2$ s.

(2)

Leave blank

Question 2 continued

Leave blank

4. A small shellfish is attached to a wall in a harbour. The rise and fall of the water level is modelled as simple harmonic motion and the shellfish as a particle. On a particular day the minimum depth of water occurs at 10 00 hours and the next time that this minimum depth occurs is at 22 30 hours. The shellfish is fixed in a position 5 m above the level of the minimum depth of the water and 11 m below the level of the maximum depth of the water. Find

(a) the speed, in metres per hour, at which the water level is rising when it reaches the shellfish,

(7)

(b) the earliest time after 10 00 hours on this day at which the water reaches the shellfish.

(4)

Leave blank

Question 4 continued

Leave blank

7.

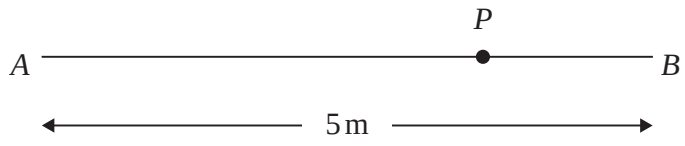


Figure 4

A and B are two points on a smooth horizontal floor, where $AB = 5$ m.

A particle P has mass 0.5 kg. One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N, are attached to P and B, as shown in Figure 4.

(a) Find the extensions in the two springs when the particle is at rest in equilibrium.

(5)

Initially P is at rest in equilibrium. It is then set in motion and starts to move towards B. In the subsequent motion P does not reach A or B.

(b) Show that P oscillates with simple harmonic motion about the equilibrium position.

(4)

(c) Given that the initial speed of P is $\sqrt{10}$ m s^{-1}, find the proportion of time in each complete oscillation for which P stays within 0.25 m of the equilibrium position.

(7)

Leave blank

Question 7 continued

Leave blank

2. A particle P moves in a straight line with simple harmonic motion of period 2.4 s about a fixed origin O. At time t seconds the speed of P is $v\,\text{m s}^{-1}$. When $t=0$, P is at O. When $t=0.4$, $v=4$. Find

(a) the greatest speed of P,

(7)

(b) the magnitude of the greatest acceleration of P.

(2)

Leave blank

Question 2 continued

Leave blank

4.

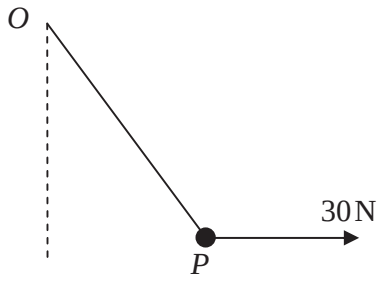


Figure 3

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 30 N is applied to P, as shown in Figure 3. The particle P is in equilibrium and the elastic energy stored in the string is 10 J.

Calculate the length OP.

(10)

Leave blank

Question 4 continued

Leave blank

2. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P. The magnitude of this force is inversely proportional to x^2.

At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a sphere of radius R.

(a) Prove that the magnitude of the gravitational force on P is $\frac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U.

(b) Find U in terms of g and R.

(7)

Leave blank

Question 2 continued

Leave blank

7. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A. A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O, vertically below A.

(a) Find the distance AO.

(3)

The particle is now raised to point C vertically below A, where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\left(\frac{a}{g}\right)}$.

(5)

It is given that $OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.

(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D, moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.

(4)

Leave blank

Question 7 continued

Leave blank

4. A particle P moves along the x-axis. At time t seconds its displacement, x metres, from the origin O is given by $x = 5 \sin (\frac{1}{3}\pi t)$.

(a) Prove that P is moving with simple harmonic motion.

(3)

(b) Find the period and the amplitude of the motion.

(2)

(c) Find the maximum speed of P.

(2)

The points A and B on the positive x-axis are such that $OA = 2$ m and $OB = 3$ m.

(d) Find the time taken by P to travel directly from A to B.

(4)

Leave blank

Question 4 continued

Leave blank

7. A particle P of mass 0.5 kg is attached to the mid-point of a light elastic string of natural length 1.4 m and modulus of elasticity 2 N. The ends of the string are attached to the points A and B on a smooth horizontal table, where $AB = 2$ m. The mid-point of AB is O and the point C is on the table between O and B where $OC = 0.2$ m. At time $t = 0$ the particle is released from rest at C. At time t seconds the length of the string AP is $(1+x)$ m.

(a) Show that the tension in BP is $\frac{2}{7}(3-10x)$ N.

(2)

(b) Find, in terms of x, the tension in AP.

(1)

(c) Show that P performs simple harmonic motion with period $2\pi\sqrt{\left(\frac{7}{80}\right)}$ s.

(6)

(d) Find the greatest speed of P during the motion.

(2)

The point D lies between O and A, where $OD = 0.1$ m.

(e) Find the time taken by P to move directly from C to D.

(4)

Leave blank

Question 7 continued

Leave blank

2. A particle P is moving in a straight line with simple harmonic motion. The centre of the oscillation is the fixed point C, the amplitude of the oscillation is 0.5 m and the time to complete one oscillation is $\frac{2\pi}{3}$ seconds. The point A is on the path of P and 0.2 m from C.

Find

(a) the magnitude and direction of the acceleration of P when it passes through A,

(3)

(b) the speed of P when it passes through A,

(2)

(c) the time P takes to move directly from C to A.

(3)

Leave blank

5. Above the Earth's surface, the magnitude of the gravitational force on a particle due to the Earth is inversely proportional to the square of the distance of the particle from the centre of the Earth. The Earth is modelled as a sphere of radius R and the acceleration due to gravity at the Earth's surface is g. A particle P of mass m is at a height x above the surface of the Earth.

(a) Show that the magnitude of the gravitational force acting on P is

$$\frac{mgR^2}{(R+x)^2}$$

(3)

A rocket is fired vertically upwards from the surface of the Earth. When the rocket is at height $2R$ above the surface of the Earth its speed is $\sqrt{\left(\frac{gR}{2}\right)}$. You may assume that air resistance can be ignored and that the engine of the rocket is switched off before the rocket reaches height R.

Modelling the rocket as a particle,

(b) find the speed of the rocket when it was at height R above the surface of the Earth.

(9)

Leave blank

Question 5 continued

Leave blank

2. A particle P moves in a straight line with simple harmonic motion about a fixed centre O. The period of the motion is $\frac{\pi}{2}$ seconds. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, P is at O and $v = 6$. Find

(a) the greatest distance of P from O during the motion,

(3)

(b) the greatest magnitude of the acceleration of P during the motion,

(2)

(c) the smallest positive value of t for which P is 1 m from O.

(3)

Leave blank

Question 2 continued

Leave blank

5. A particle P is moving in a straight line with simple harmonic motion on a smooth horizontal floor. The particle comes to instantaneous rest at points A and B where AB is 0.5 m. The mid-point of AB is O. The mid-point of OA is C. The mid-point of OB is D. The particle takes 0.2 s to travel directly from C to D. At time $t = 0$, P is moving through O towards A.

(a) Show that the period of the motion is $\frac{6}{5}$ s.

(5)

(b) Find the distance of P from B when $t = 2$ s.

(3)

(c) Find the maximum magnitude of the acceleration of P.

(2)

(d) Find the maximum speed of P.

(2)

Leave blank

Question 5 continued

Leave blank

7. Two points A and B are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point A. A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point B. A particle P of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point C on the straight line between A and B.

(a) Show that the length of AC is 1.76 m.

(4)

The particle P is now held at the point D on the line AB such that $AD = 2.16$ m. The particle is then released from rest and in the subsequent motion both strings remain taut.

(b) Show that P moves with simple harmonic motion.

(4)

(c) Find the speed of P as it passes through the point C.

(2)

(d) Find the time from the instant when P is released from D until the instant when P is first moving with speed 2 m s^{-1}.

(4)

Leave blank

Question 7 continued

Leave blank

6.

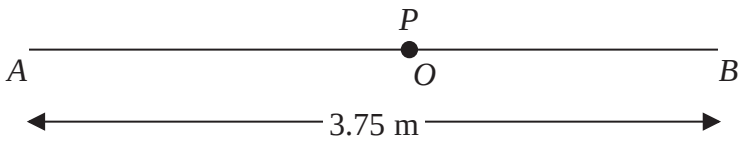


Figure 5

The points A and B are 3.75 m apart on a smooth horizontal floor. A particle P has mass 0.8 kg. One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N, are attached to P and B. The particle P rests in equilibrium at the point O, where AOB is a straight line, as shown in Figure 5.

(a) Show that $AO = 2.4$ m.

(4)

The point C lies on the straight line AOB between O and B. The particle P is held at C and released from rest.

(b) Show that P moves with simple harmonic motion.

(5)

The maximum speed of P is $\sqrt{2}$ m s^{-1}.

(c) Find the time taken by P to travel 0.3 m from C.

(5)

Leave blank

Question 6 continued